

# Fast magnetization in counterstreaming plasmas with temperature anisotropies

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## Abstract

Counterstreaming plasmas exhibits an electromagnetic unstable mode of filamentation type, which is responsible for the magnetization of plasma system. It is shown that filamentation instability becomes significantly faster when plasma is hotter in the streaming direction. This is relevant for astrophysical sources, where strong magnetic fields are expected to exist and explain the nothermal emission observed.

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## I. INTRODUCTION

Presently, there is an increasing interest for a correct understanding of purely growing electromagnetic instabilities driven by a velocity anisotropy of plasma particles. [1, 2, 3, 4, 5, 6]. These instabilities will release the excess of perpendicular free energy stored in the particle velocity anisotropy, whether it is a temperature anisotropy [7] or counterstreaming plasmas [8]. A substantial fraction of the kinetic energy of plasma particles is transformed by the instability and contribute to the amplification of magnetic energy [9].

Counterstreaming plasma structures are present in laboratory experiments and astrophysical systems, and they are widely investigated either to prevent the unstable modes arising in beam-plasma experiments [10, 11], or to prove the existence of large scale magnetic fields in astrophysical objects [12, 13, 14, 15].

The spontaneous magnetization of counterstreaming plasmas is associated with filamentation instability [8], which is described as a counterstreaming based electromagnetic mode purely growing in time and propagating perpendicular to the streams. Here we show that the magnetization process associated with filamentation instability can be markedly faster in counterstreaming plasmas with temperature anisotropies, namely, when plasma is hotter along streaming direction. This fact can be decisive for the potential role of filamentation instability in producing strong quasistatic magnetic fields in astrophysical plasmas [9, 13]. As long the maximum growth rate of filamentation mode is much larger than the growth rates of all other plasma instabilities (e.g., two-stream electrostatic instability), then the filamentation instability will be fastest, and it will be the primary mechanism for the relaxation of the initial counterstreaming configuration [14].

## II. COUNTERSTREAMING PLASMAS WITH TEMPERATURE ANISOTROPIES

The oscillatory properties of a plasma are determined by the dielectric tensor,  $(\epsilon)$ , and using the linearization of Vlasov-Maxwell equations we simply get [16]

$$(\epsilon) = \mathbf{I} + \sum_a \frac{\omega_{p,a}^2}{\omega^2} \left[ \int_{-\infty}^{+\infty} d\mathbf{v} \, \mathbf{v} \frac{\partial f_{a,0}}{\partial \mathbf{v}} + \int_{-\infty}^{+\infty} d\mathbf{v} \frac{1}{\omega - \mathbf{k} \cdot \mathbf{v}} (\mathbf{k} \cdot \frac{\partial f_{a,0}}{\partial \mathbf{v}}) \mathbf{v} \mathbf{v} \right]. \quad (1)$$

where the unperturbed velocity distribution function  $f_{a0}(\mathbf{v})$  of the particles of sort  $a$  is normalized by  $\int d\mathbf{v} f_{a0}(\mathbf{v}) = 1$ ,  $\omega_{p,a} = (4\pi n_a e^2 / m_a)^{1/2}$  is the plasma frequency,  $\mathbf{I}$  is the unity tensor, and  $\omega$  and  $k$  are respectively, the frequency and the wave number of plasma modes.

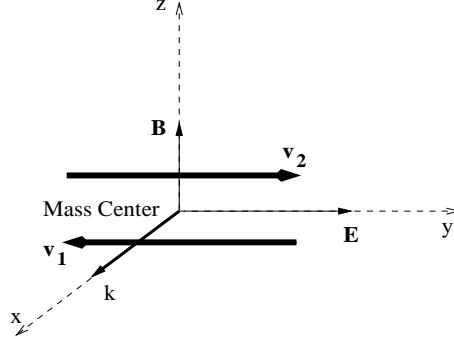


FIG. 1: Two counterstreaming plasmas and the electromagnetic filamentation mode with fields  $\mathbf{E}$  and  $\mathbf{B}$ , and the wavevector  $\mathbf{k}$  perpendicular to the streams.

In Figure 1 we fix the orientation for the counterstreams and the filamentation mode: the electric field,  $\mathbf{E} \parallel \mathbf{v}_{1,2}$ , is along the streaming direction, and the wave vector,  $\mathbf{k} \perp \mathbf{v}_{1,2}$ , is perpendicular to the streams. This electromagnetic mode will be solution of the following dispersion equation

$$\frac{k^2 c^2}{\omega^2} = \epsilon_{yy}. \quad (2)$$

with  $\epsilon_{yy}$  given by (1), and  $c$  is the speed of light in vacuum.

For the sake of simplicity, we assume that the counterstreams are symmetric with equal intensities, and moving with the same velocities,  $|v_1| = v_2 = v_0$ . Moreover, the effect of ions is minimized to a positive background, and the electron plasma counterstreams are assumed to be homogeneous, and charge and current neutralized.

### A. Moderate thermal effects

We first suppose that only one of the counterstreaming plasmas exhibits a temperature anisotropy of a bi-Maxwellian type, i.e. with two characteristic thermal velocities,  $v_{th,x} = v_{th,z} = v_{th} < v_{th,y}$ , and the other one is monochromatic (cold)

$$f_0(v_x, v_y, v_z) = \frac{1}{2\pi^{3/2} v_{th}^2 v_{th,y}} e^{-\frac{v_x^2 + v_z^2}{v_{th}^2}} e^{-\frac{(v_y + v_0)^2}{v_{th,y}^2}} + \frac{1}{2} \delta(v_x) \delta(v_y - v_0) \delta(v_z). \quad (3)$$

We substitute (3) in (1), and then for dispersion relation (2) we find

$$\frac{k^2 c^2}{\omega^2} = 1 - \frac{\omega_{pe}^2}{2\omega^2} \left(1 + \frac{k^2 v_0^2}{\omega^2}\right) - \frac{\omega_{pe}^2}{2\omega^2} \left\{1 + \left[\frac{1}{2}(A+1) + \left(\frac{\beta_0}{\beta_{th}}\right)^2\right] Z'\left(\frac{\omega}{kv_{th}}\right)\right\}, \quad (4)$$

where  $\beta_{th} = v_{th}/c$ ,  $\beta_0 = v_0/c$ , and  $Z'(f)$  is the derivative of the well-known plasma dispersion function [17]

$$Z(f) = \pi^{-1/2} \int_{-\infty}^{\infty} dx \frac{\exp(-x^2)}{x-f}, \quad f = \frac{\omega}{kv_{th}}. \quad (5)$$

We assume that plasma is hotter in the streaming direction,  $v_{th,y} > v_{th}$ , which defines here a positive temperature anisotropy  $A_1 = A = (v_{th,y}/v_{th})^2 - 1 > 0$ . In this case the instability is driven equally by the counterstreaming motion of plasma and by the temperature anisotropy of plasma particles, and therefore we should achieve an enhancing effect for the growth rates of filamentation mode.

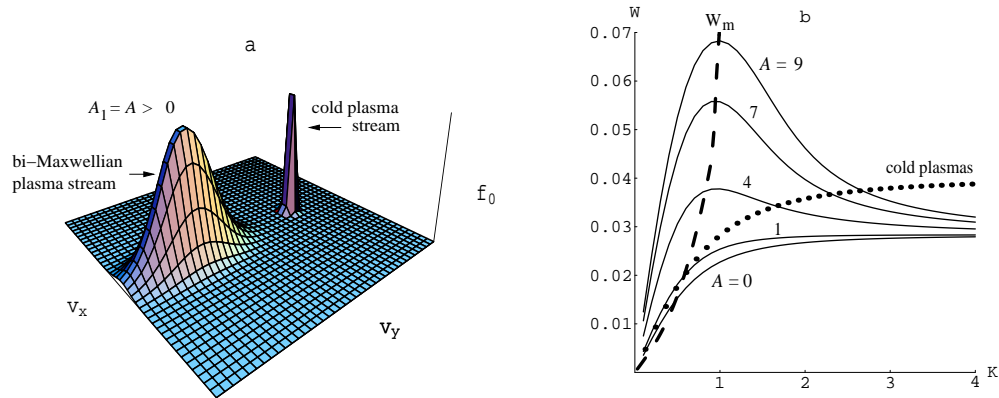


FIG. 2: (a) Schematic representation for the distribution functions of two counterstreaming plasmas: one is assuming cold, and the other one with a two-temperatures anisotropy; (b) The growth rates of filamentation mode (solid lines) for  $\beta_{th} = 0.1$ ,  $\beta_0 = 0.04$ , and different values of thermal anisotropy:  $A = 9, 7, 4, 1, 0$ . The growth rates are also plotted for two cold counterstreaming plasmas (dotted line). The dashed line represents the *locus* of the maximum growth rates. The coordinates are scaled as  $W = \Omega/\omega_{pe}$  and  $K = kc/\omega_{pe}$ .

The aperiodic solutions of (4),  $\Im(\omega) = \Omega = \Omega(k)$  ( $\Re(\omega) = 0$ ), are plotted in Figure 2 (b) with solid lines for different values of the positive anisotropy  $A = 0, 1, 4, 7, 9$ , and with dotted line when both counterstreaming plasmas are cold. In this case, thermal effects are

sufficiently low and the so-called "cold anisotropy" determined by the the relative motion of plasmas is still dominant leading to a mixture of the features from both filamentation and Weibel modes. Therefore, for small wavenumbers the aperiodic solutions are characterized by a maximum,  $\Omega_m = \Omega_m(k_m)$  and the corresponding wavenumber  $k_m$  (Weibel regime), and for very large wavenumbers,  $k \rightarrow \infty$ , they approach an asymptotic value,  $\Omega_a = \Omega(k \rightarrow \infty)$  (filamentation regime). The last one is simply found as being lower than the asymptotic value obtained for the filamentation growth rate when the both counterstreaming plasmas are cold,  $\Omega_a^{\text{cold}}$ ,

$$\Omega_a = \frac{\sqrt{2}}{2} \frac{\omega_{pe} v_0}{c} < \Omega_a^{\text{cold}} = \frac{\omega_{pe} v_0}{c}. \quad (6)$$

Knowing the maximum values for the growth rates of filamentation mode is essential for a correct evaluation of the magnetization process in laboratory or astrophysical applications [13, 18]. If the filamentation mode has the largest maximum growth rate comparing to the other plasma instabilities (e.g., two-stream electrostatic instability), then it will be the fastest mechanism by which is released the free energy stored into the initial motion of plasma counterstreams.

The aperiodic solutions plotted in Fig. 2 (b), are sensitive to values close to unity for the arguments of plasma dispersion function in (4)–(5). Therefore, we keep the accuracy of a general approach, showing that the maximum growth rate,  $\Omega_m$ , can be determined exactly numerically without any restriction to large or small arguments of  $Z(f)$ . The maximum growth rate,  $\Omega_m$ , depends on the temperature anisotropy (see in Fig. 2) and it is given in (4) by the following condition

$$\frac{d\Omega}{dk} = 0. \quad (7)$$

But the maximum value,  $\Omega_m$ , is solution of (4) as well, and eliminating the anisotropy from equations (4) and (7) we simply find [16]

$$\left[ 1 + \frac{i\Omega_m}{k_m v_{th}} Z\left(\frac{i\Omega_m}{k_m v_{th}}\right) \right] \left[ M_1(\Omega_m, k_m) \left( \frac{2\Omega_m^2}{k_m^2 v_{th}^2} + 3 \right) - 4 \right] = M_1(\Omega_m, k_m), \quad (8)$$

where

$$M_1(\Omega_m, k_m) = \frac{2k_m^2 c^2}{\omega_{pe}^2} - \frac{k_m^2 v_0^2}{\Omega_m^2} + 2. \quad (9)$$

This is an implicit form equation for the maximum growth rates,  $[k_m, \Omega_m]$ , which we plot with dashed line in Fig. 2 (b). Intersections of the dashed line with dispersion curves given by (4) and plotted with solid lines in Fig. 2 (b), will give us the maximum values corresponding to different temperature anisotropies,  $\Omega_m = \Omega_m(k_m, A)$ .

We remark that only the large wave-lengths will be affected by the temperature effects (the Weibel-like regime) where the amplitude is growing significantly faster. At saturation, the growth rates can be markedly larger than those obtained for isotropic counterstreams ( $A = 0$ ), or for cold plasmas. This enhancing effect is diminished at small wave-lengths, where the growing mode is not affected by the temperature anisotropy.

## B. Strong thermal effects

We assume now that both plasma counterstreams are dominated by the thermal effects, with bi-Maxwellian distributions of positive anisotropies,  $A_{1,2} = (v_{th,y1,2}/v_{th})^2 - 1 \geq 0$ . In this case the counterstreams are modeled by the following distribution function

$$f_0(v_x, v_y, v_z) = \frac{1}{2\pi^{3/2}v_{th}^2 v_{th,y}} e^{-\frac{v_x^2 + v_z^2}{v_{th}^2}} \left[ e^{-\frac{(v_y + v_0)^2}{v_{th,y,1}^2}} + e^{-\frac{(v_y - v_0)^2}{v_{th,y,2}^2}} \right], \quad (10)$$

which is schematically presented in Fig. 3 (a), and substituted in (1) yields the following dispersion relation

$$\frac{k^2 c^2}{\omega^2} = 1 - \frac{\omega_{pe}^2}{\omega^2} \left\{ 1 + \frac{1}{2} \left[ 1 + \frac{1}{2}(A_1 + A_2) + 2 \left( \frac{\beta_0}{\beta_{th}} \right)^2 \right] Z' \left( \frac{\omega}{kv_{th}} \right) \right\}. \quad (11)$$

Due to the strong temperature effects the aperiodic solutions from (11),  $\Omega(k) \geq 0$ , appear to resemble the Weibel regime, see the solid line curves in Fig. 3 (b), and the unstable mode does not exist for  $k \geq k_c$ , where the cutoff wave number  $k_c$  is the nontrivial solution of (8) to the limit of  $\Omega = 0$

$$k_c(A_1, A_2) = \frac{\omega_{pe}}{c} \left[ \frac{1}{2}(A_1 + A_2) + 2 \left( \frac{\beta_0}{\beta_{th}} \right)^2 \right]^{1/2}. \quad (12)$$

For an exact evaluation of the maximum growth rate,  $[k_m, \Omega_m]$ , we invoke again condition (7) and for the dispersion relation (11) we derive as before, an implicit form equation

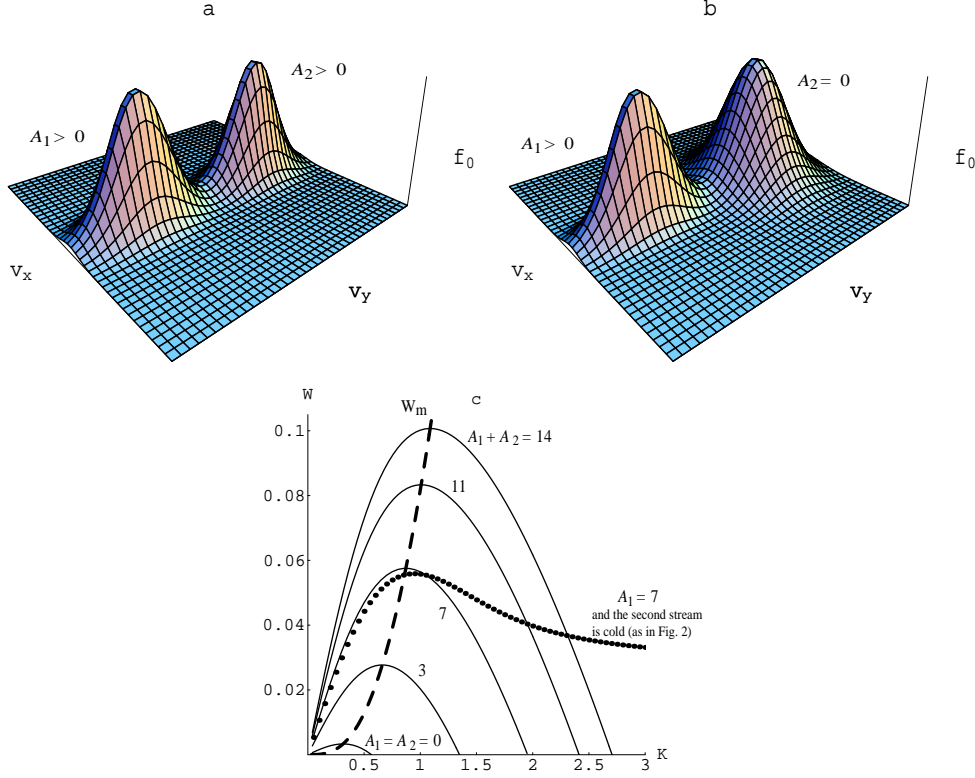


FIG. 3: (a) and (b) Schematic representation for the distribution functions of counterstreaming plasmas dominated by thermal effects. (c) With solid lines are shown the aperiodic solutions of (11) for  $\beta_{th} = 0.1$ ,  $\beta_0 = 0.04$  and different temperature anisotropies  $A_1 + A_2 = 14, 11, 7, 3$ , or  $A_1 + A_2 = 0$ . The dashed line represents the *locus* of the maximum growth rates. The coordinates are scaled as in Fig. 2.

$$\left[1 + \frac{i\Omega_m}{k_m v_{th}} Z\left(\frac{i\Omega_m}{k_m v_{th}}\right)\right] \left[M_2(\Omega_m, k_m) \left(\frac{2\Omega_m^2}{k_m^2 v_{th}^2} + 1\right) + 2\right] = M_2(\Omega_m, k_m), \quad (13)$$

with

$$M_2(\Omega_m, k_m) = \frac{\omega_{pe}^2}{k_m^2 c^2} + \frac{\Omega_m^2}{k_m^2 c^2} + 1. \quad (14)$$

The maximum growth rates from (13) are plotted with dashed line in Fig. 3 (c), and the intersections with dispersion curves given by (11) and plotted with solid lines, will give us the maximum values corresponding to different temperature anisotropies,  $\Omega_m = \Omega_m(k_m, A)$ . In this case thermal effects limit the existence of the unstable mode to large wave-lengths,  $k \leq k_c$ , where  $k_c$  is given by (12). This limit is removed,  $k_c \rightarrow \infty$ , only for  $\beta_{th} \rightarrow 0$ . However, the growth rates can be orders of magnitude larger as long the positive temperature

anisotropies are sufficiently large.

### III. CONCLUSIONS

We have investigated the purely growing electromagnetic mode that arises in counterstreaming plasmas and propagates perpendicular to the streaming direction. This instability is responsible for the magnetization of such plasma systems. We have generalized the theoretical approach assuming that counterstreaming plasmas exhibit positive temperature anisotropies of a bi-Maxwellian type. In this case the instability is driven equally by the free energy stored in the counterstreaming motion of plasma and the temperature anisotropy of plasma particles. Consequently, we found that the aperiodic modes can reach maximum growth rates with order of magnitudes larger than those calculated for cold plasmas or for counterstreaming plasmas with an isotropic temperature distribution. Thus, the filamentation mode is enhanced by the positive temperature anisotropies (i.e., when plasma is hotter in the streaming direction), and it can become faster than all the other plasma instabilities. This effect improves the efficiency of magnetic field generation, and gives further support for the potential role of magnetic instabilities in the fast magnetization applications.

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